Bose glass and superfuid phases of cavity polaritons

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We report the calculation of cavity exciton-polariton phase diagram which takes into account the presence of realistic structural disorder. Polaritons are modelled as weakly interacting two-dimensional bosons. We show that with increasing density polaritons first undergo a quasi-phase transition toward a Bose glass: the condensate is localized in at least one minimum of the disorder potential, depending on the value of the chemical potential of the polariton system. A further increase of the density leads to a percolation process of the polariton fluid which gives rise to a Kosterlitz-Thouless phase transition towards superfluidity. The spatial representation of the condensate wavefunction as well as the spectrum of elementary excitations are obtained from the solution of the Gross-Pitaevskii equation for all the phases.

Exciton-polaritons (polaritons) in microcavities are composite 2-dimensional weakly interacting bosons [1, 2]. Despite their short radiative life time ($\sim 10^{-12}$ s) stimulated scattering towards their ground state has been demonstrated [3, 4], and quasi-thermal equilibrium was recently reported [5]. In this quasi-equilibrium regime cavity polaritons are expected to give rise to a Kosterlitz-Thouless (KT) phase transition toward superfluidity [6]. The corresponding phase diagram has been established a few years ago [7], and recently refined to fully take into account the non-parabolic shape of the polariton dispersion [8]. Because of their light effective mass (typically 10^{-4} times the free electron mass) polaritons show extremely small critical density and high critical temperatures that can be larger than room temperature in some cases. However, semiconductors were assumed to be ideal in the approach used in Refs. [7] and [8] while experimental data clearly show strong localization of the condensate because of structural imperfections. The phase observed is in fact characteristic of a Bose glass [9] and no signature of superfluidity has been reported thus far. In this Letter we propose the derivation of a new polariton phase diagram taking into account structural disorder whose impact on the spatial shape of the wavefunction and the dispersion of elementary excitations, is analyzed within the framework of the Gross-Pitaevskii theory [10].

To give a qualitative picture of the model, we assume that the polaritons are moving in a random potential $V(\mathbf{r})$ whose mean amplitude and root mean square fluctuation are given by $\langle V(\mathbf{r})\rangle = 0$ and $\sqrt{\langle V^2(\mathbf{r})\rangle} = V_0$ respectively. The correlation length of this potential is $R_0 = \sqrt{\int \langle V(\mathbf{r})V(0)\rangle \mathrm{d}\mathbf{r}/V_0^2}$. As in any disordered system, there are here two types of polaritons states [11]: the free propagating states and the localized states with energy $E < E_c$, where E_c is the critical "delocalization" energy. The localization radius scales like $a(E) \propto a_0 V_0^s/(E_c - E)^s$, s being a critical index and

$a_0 = \sqrt{\hbar^2/mV_0}$ [12]. In two dimensions E_c is of the order of mean potential energy (i.e. 0 in our case), and $s \approx 0.75$ [11]. The quasi-classical density of states is $D_0(E) \cong M/4\pi\hbar^2[1 + \mathrm{erf}\,(E/V_0)]$ [13]. However, an exciton-polariton is a composite boson also containing a photonic component which makes its trapping in the state with the localization radius $a = a_c \lesssim \lambda$ (where λ is the wavelength of the incident light) not possible. Thus the resulting effective density of states shows an abrupt cut-off at $E = E_0$ which is self-consistently determined from $a(E_0) = a_c$: $D(E) = D_0(E)$ for $E > E_0$ and $D(E) = 0$ otherwise.

Clearly, non-interacting bosons cannot undergo Bose Einstein Condensation (BEC) as the number of particles which can be fitted to all the excited states of the system ($E > E_0$) is divergent. The situation is thus different from those for cold atoms trapped by power-low potential in 2D, where the renormalization of the density of states makes "true" BEC possible [14]. Therefore, even in the presence of disorder, BEC cannot take place strictly speaking for cavity polaritons. However, it is possible to define a quasi-phase transition which takes place in finite systems [7]. Indeed, for the finite size R system there is a finite number N_{trap} of potential traps for polaritons, thus, there is an energy spacing between the single particle states. The typical energy distance between the ground and excited states of the finite-size system levels δ under the assumption of long-range potential is approximately given by V_0/N_{trap} or $\hbar^2/2MR_0^2$ whichever is smaller. In this framework the critical density is given by the total number of polaritons which can be accomodated in all the energy levels of the disorder potential $V(\mathbf{r})$ except the ground one [7]:

$$n_c(T) = \sum_{i \neq 0} f_B(E_i, E_0, T), \tag{1}$$

where $f_B(T, \mu, T)$ is the Bose-Einstein distribution func-

tion, μ is the chemical potential.

To evaluate the critical density $n_c(T,R)$, the discrete sum is replaced by an integral in Eq. (1), and we find $n_c(T) \approx D(E_0)k_B T \ln\left[1/(1-e^{\delta/k_B T})\right]$ assuming $D(E)$ is a smooth function. Above this density all additional particles are accumulating in the ground state and the concentration of condensed particles n_0 satisfies $n_0 \geq (n - n_c)$, where n is the total density of polaritons. It is not a real phase transition since the system has a discrete energy spectrum and the value of the chemical potential never becomes strictly equal to E_0.

Interactions between particles start to become dominant once the polaritons start to accumulate in the ground state. The situation can be qualitatively described as follows: particles start to fill the lowest energy state which is therefore blue shifted because of interactions ($\mu - E_0 > 0$). Thus, for some occupation number of the condensate the chemical potential reaches the energy of another localized state, and this state starts in turn to populate and to blue shift. The condensate, like a liquid, fills several minima of the potential. It gives rise to the spatial and reciprocal space pictures of Refs. [5, 6]. A few localized states, covering about 20 % of the surface of the emitting spot are all emitting light at the same energy and are strongly populated. This characterizes a Bose glass [9]. This situation occurs up to the achievement of the condition $\mu = E_c$. This condition should be accompanied by a percolation of the condensate which at this stage should cover 50 % or more of the sample (in the semiclassical representation). The delocalized condensate becomes at this stage a KT superfluid. More precisely, the different sides of the finite-size system are linked by the phase coherent path. Therefore we predict *two* quasi-phase transitions driven by temperature and particle density: first, with an increase of the polariton density beyond $n_c(T)$ the system enters the Bose glass phase, then with a further increase of the density the polariton system becomes superfluid. The critical condition $\mu = E_c$ is valid only at low temperature where the thermal depletion of the condensate is negligible.

The quantitative analysis can be carried out in the framework of the Gross-Pitaevskii equation for the condensate wavefunction $\Psi(\mathbf{r},t)$ which reads

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left(-\frac{\hbar^2}{2M}\triangle + V(\mathbf{r}) + g\left|\Psi(\mathbf{r},t)\right|^2\right)\Psi(\mathbf{r},t), \quad (2)$$

where g is a constant characterizing the weak repulsive interaction between polaritons. The solution of the Gross-Pitaevskii equation takes the form $\Psi(\mathbf{r},t) = \Psi_0(\mathbf{r})\exp\left(-i\mu t/\hbar\right)$. Top panels (a), (b), and (c) of Fig. 1 show the real space distribution of the polaritons obtained from the solution of the Gross-Pitaevskii equation. The parameters are those of a realistic CdTe microcavity at zero detuning. We have taken the polariton mass
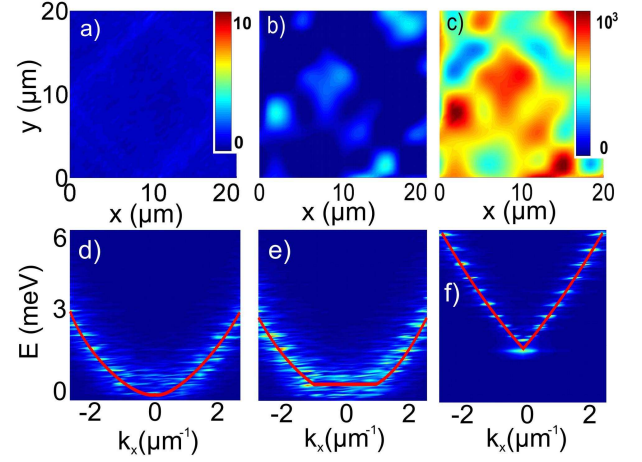

FIG. 1: (Color online) Spatial images (top panels) and quasiparticle spectra (bottom panels) for a realistic disorder potential. The figures shown correspond to densities 0, 6×10^{10} and 2×10^{12} cm^{-2}. The red lines are only guides to the eye, showing parabolic, flat and linear-type dispersions. The colormap of the panel 1.b is the same as 1.c.

$m = 5 \times 10^{-5}m_0$, where m_0 is the free electron mass, and the interaction constant $g = 3E_b a_B^2/N_{qw}$, where E_b is the exciton binding energy (25 meV in CdTe), $a_B = 34$ Å is the exciton Bohr radius and $N_{qw} = 16$ is the number of wells embedded in the microcavity. We have included a random disorder potential with $V_0 = 0.5$ meV and $R_0 = 3$ μm. Figure 1.a corresponds to the non-condensed situation. The spatial profile is given by the statistical averaging over the all occupied states, $n(\mathbf{r}) = \sum_j f_B(E_j, T, \mu(T))|\Psi_j(\mathbf{r})|^2$. Here the temperature is set to $T = 19$ K which corresponds to the effective polariton temperature measured in [5]. In this case the total number of particles is small and thus nonlinear terms in Gross-Pitaevskii equation can be neglected.

Once the quasi-condensate is formed, and for moderate temperatures, one can neglect the thermal occupation of the excited states and the spatial image of the polariton distribution is given by the ground state wavefunction which corresponds to solution of Eq. (2). We show the resulting density below and above the percolation threshold on Figs. 1.b and 1.c respectively. As expected, the condensate is localized in a few minima of the random potential as shown on Fig. 1.b. On Fig. 1.c the condensate wave function still exhibits some spatial fluctuations connected to disorder, but the condensate is nonetheless well delocalized, covering the whole sample area.

To calculate the quasiparticle spectra shown in lower panels 1.d, 1.e and 1.f of Fig. 1 we introduce a single-particle Green's function which takes the form

$$G_\omega(\mathbf{r},\mathbf{r}_0) = \sum_j \frac{\Psi_j(\mathbf{r})\Psi_j^\dagger(\mathbf{r}_0)}{\hbar\omega - E_j}, \quad (3)$$

where E_j and $\Psi_j(\mathbf{r})$ are energies and eigenfunctions of the elementary excitations [15], found numerically from Eq. (2). The spectrum of elementary excitations is given by the poles of the Green's function in the $(\mathbf{k}, \omega)$ representation, and shown on the lower panels of Fig. 1. The left panel 1.d shows typical parabolic dispersion broadened by the disorder potential. The middle panel 1.e shows parabolic dispersion with a flat part produced by the localization of the condensate. The linear spectrum on the right panel 1.f is the distinct feature of the superfluid state of the system. Only the upper Bogoliubov branch is shown. Figures 1.b and 1.e reproduce quite well the experimental observations of Ref. [5] which are characteristics of the formation of a Bose glass.

It is instructive to analyze both the variation of the emission pattern and the quasi-particle spectrum in comparison with the behavior of the superfluid fraction of the polariton system. The latter quantity can be calculated using the twisted boundary conditions method [16]. Imposing such boundary conditions implies that the condensate wavefunction acquires a phase between the boundaries, namely

$$\Psi_\theta(\mathbf{r} + \mathbf{L}_i) = \mathrm{e}^{i\theta}\Psi_\theta(\mathbf{r}), \tag{4}$$

where $\mathbf{L}_i$ $(i = x, y)$ are the vectors which form the rectangle confining the polaritons and θ is the twisting parameter. The superfluid fraction of the condensate is given by [16]

$$f_\mathrm{s} = \frac{n_\mathrm{s}}{n} = \lim_{\theta \to 0} \frac{2ML^2(\mu_\theta - \mu_0)}{\hbar^2 n\theta^2}, \tag{5}$$

where μ_θ is the chemical potential corresponding to the boundary conditions Eq. (4) and μ_0 is the chemical potential corresponding to the periodic boundary conditions ($\theta = 0$). In the case of a clean system, $V(\mathbf{r}) = 0$, the plane wave is the solution of Eq. (2) and $\mu_\theta - \mu_0 = n\hbar^2\theta^2/2ML^2$: the superfluid fraction is $f_\mathrm{s} = 1$. On the contrary, for the strongly localized condensate the wavefunction is exponentially small at the system boundaries and the change of the boundary condition (i.e. variation of θ) does not change the energy of the system, thus $f_\mathrm{s} \sim \exp\left[-L/a(\mu_0)\right]$ and goes to 0 for the infinite system. Due to the exponential tails of the localized wavefunctions a small degree of superfluidity remains in the finite size system. Equations (2) and (4) allow to study the depletion of the superfluid fraction for arbitrary disorder. The contribution of the disorder to the normal density of polaritons can be represented as

$$n_\mathrm{n}^\mathrm{d} = (1 - f_\mathrm{s})n. \tag{6}$$

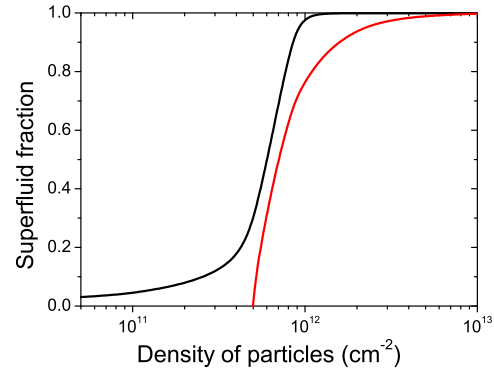Figure 2 shows the superfluid fraction calculated as a function of the polariton density in the system for

FIG. 2: (Color online) Superfluid fraction as a function of the density of particles, obtained from twisted boundary conditions (black curve) and from the perturbative approach (red curve).

$T = 0$ K. Due to the finiteness of the system considered the superfluid fraction remains non-zero for any finite density, but a very clear threshold behavior for densities corresponding to the percolation threshold as observed on Fig. 1, is also shown. For high values of the chemical potential, where $V_0^2/\mu g \ll 1$, perturbation theory applies and we obtain

$$n_\mathrm{n}^\mathrm{d} = \frac{1}{4}\frac{V_0^2}{\mu g}, \tag{7}$$

for the normal density [17]. The resulting curve is shown in red on Fig. 2. The twisted boundary conditions approach and Eq. (7) give coinciding results for high density of the polaritons.

In the rest of the paper, we concentrate on the establishment of the cavity polariton phase diagram. Similarly to previous works [7], we roughly define a temperature and density domain where the strong coupling is supposed to hold. The limits are shown on Fig. 3 as thick dotted lines [20]. The transition from normal to Bose glass phase can be calculated from Eq. (1) and a realistic realization of disorder. The lower solid line on Fig. 3 shows $n_c(T)$ for the same realization of disorder as for Fig. 1. The free polariton dispersion is calculated using the geometry of Ref. [5]. At $T = 19$ K we find $n_c = 2 \times 10^8 \mathrm{cm}^{-2}$.

We now calculate the density for the transition between the Bose glass and the superfluid phase. In the low temperature domain, this density is approximately given by the percolation threshold $\mu = E_c$ and does not depend significantly on temperature. This condition corresponds with good accuracy to the abrupt change of the superfluid fraction f_s shown in Fig. 2. However, at higher temperature the thermal depletion of the condensate becomes the dominant effect. In that case the chemical

potential of the condensate is much higher than the percolation energy E_c and the depletion induced by disorder can be neglected compared to the thermal depletion of the superfluid. The normal density then reads

$$n_n^0(T) = -\frac{2}{(2\pi)^2} \int E(\mathbf{k}) \frac{\partial f_B\left[\epsilon(\mathbf{k}), \mu = 0, T\right]}{\partial \epsilon} \, d\mathbf{k}, \quad (8)$$

and the superfluid density in the system is given by

$$n_s(T) = n - n_n^0(T), \quad (9)$$

which can be substituted into the Kosterlitz-Nelson formula [18] to obtain a self-consistent equation for the transition temperature:

$$T_{KT} = \frac{\hbar^2 \pi n_s(T_{KT})}{2M}. \quad (10)$$

The joint solution of Eqs. (10) and Eq. (9) allows to determine the superfluid phase transition temperature $T_{KT}(n)$. The result of this procedure is shown on Fig. 3. Below 120 K the critical density is given by the percolation threshold and there is no temperature dependence. Above 200 K the superfluid depletion is determined solely by the thermal effects. In the intermediate regime the crossover between the thermal and disorder contributions takes place and our approximations are no longer justified. We also find that the superfluid transition takes place very close to the weak to strong coupling threshold and for densities 3 orders of magnitude larger than the one of the Bose glass transition at 19 K. This suggests that experimental observation of this phenomenon remains a great challenge.

In conclusion, we have established the phase diagram of cavity polaritons taking into account the effect of structural imperfections. We predict that with increasing density the polariton system first enters the Bose glass phase before it becomes superfluid. The Bose glass picture is in good agreement with recent experimental data [5]. The condensate wavefunctions as well as the spectra of elementary excitations are obtained from the solutions of the Gross-Pitaevskii equation including disorder. Our work also shows that the presence of disorder has no significant impact on the occurence of a bosonic phase transition for polaritons. This explains why this phenomenon has been observed in a rather disordered system like CdTe. This also gives good hope for the observation of such phase transition in even more disordered systems like GaN [19]. However, since disorder strongly affects the occurence of the superfluid phase transition, it could bring renewed interest in cleaner systems like GaAs based structures.
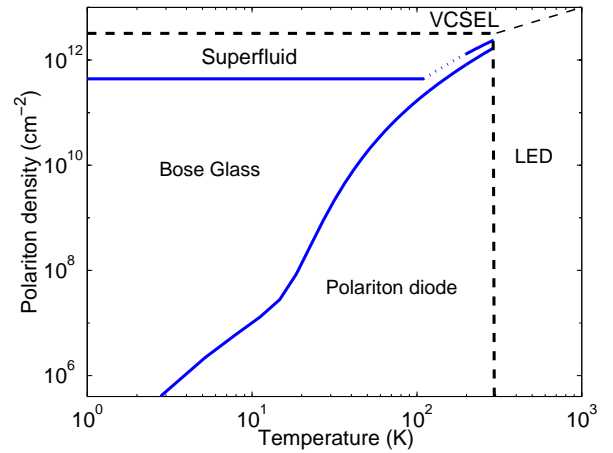
FIG. 3: (Color online) Polariton phase diagram for a CdTe microcavity containing 16 QWs. The horizontal and vertical dashed lines show the limiting temperatures and densities where the strong coupling holds. The lower solid line show the critical density for the transition from normal to Bose glass phase. The upper solid line shows the critical density for the transition from the Bose glass to the superfluid phase. The dashed part of the line shows the temperature range where the validity of our approximations ceases.

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